Transient on-off intermittency in a coupled map lattice system

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(Received 6 September 1995)

Transient behaviors are very rich and diverse in a coupled map lattice system. We find that some transient processes exhibit a characteristic on-off intermittency. The average distribution of the laminar phases for different system sizes is calculated numerically. This distribution is characterized by a power law with the exponent $-\frac{3}{2}$.

PACS number(s): 05.45.+b

As we study nonlinear complex systems, we usually pay attention to their asymptotic behaviors. In order to settle into a final attractor, one always has to throw away a certain number of iterates of periods, i.e., the transient process, while carrying out the numerical work. However, transient behavior certainly plays a more constructive role in our understanding of complex systems. In the space of a complex system, besides trivial and strange attractors, there may exist many unstable objects (periodic orbits) that are responsible for the transient behaviors in the system. Transitions among these objects, before the system settles down at an asymptotic attractor, may give rise to a rich variety of transient phenomena. In some cases, the transient process is extremely long, and one can observe only irregular and chaotic motion in finite long time, although the final behavior may be a simple attractor. In the past decade, many investigations have been shifted to the study of transient chaos in temporal and spatiotemporal systems [1–7]. The spatiotemporal systems not only exhibit very rich phenomenology including a wide variety of both spatial as well as temporal periodic structures, intermittency, chaos, solitons, frozen random patterns, periodic patterns, traveling wave, domain walls, kink dynamics, developed turbulence, etc., but also exhibit rich and diverse transient behaviors [5–15]. A characteristic of these transient behaviors is that the transient lifetime increases exponentially as the system size increases [5,6]. The geometric mechanism that is responsible for these supertransient behaviors has been addressed [7]. Recently, a very interesting phenomenon, on-off intermittency, is discussed in the temporal system [16–19], and the coupled map lattice model (CML) [20], which is introduced as a simple model with the essential features of spatiotemporal systems. In this paper we focus our attention on the study of transient on-off intermittency phenomena in CML.

Specifically, we use the following nearest-neighbor (diffusive) coupled map lattice model:

$$x_{n+1}(i)=(1-\epsilon)f(x_n(i)) + \frac{\epsilon}{2}[f(x_n(i-1))+f(x_n(i+1))] , \qquad (1)$$

where, n, i, and ϵ are the discrete time step, the lattice site index, and the coupling coefficient, respectively. The mapping function $f(x)$ is defined as the logistic map $f(x)=ax(1-x)$. We use a periodic boundary condition $x_n(i)=x_n(i+L)$, with L being the system size.

With the variation of parameters a and ϵ, the system (1) can exhibit these rich phenomena mentioned above. For instance, after the transient process, the asymptotic behavior of the system is a stable spatiotemporal period-2 solution (S2T2) shown in Fig. 1 at the parameters $a=4$, $\epsilon=0.15$, and $L=100$. The initial conditions of all sites are randomly chosen in the interval [0,1] throughout this paper. As L is even, the system always possesses the S2T2 state. But its basin becomes very narrow, and the average transient lifetime increases rapidly, and exhibits supertransient behavior as L becomes large. In Ref. [6], it was numerically found that the average transient lifetime of the S2T2 state increases exponentially with the system size L. When L (even) is large, this supertransient

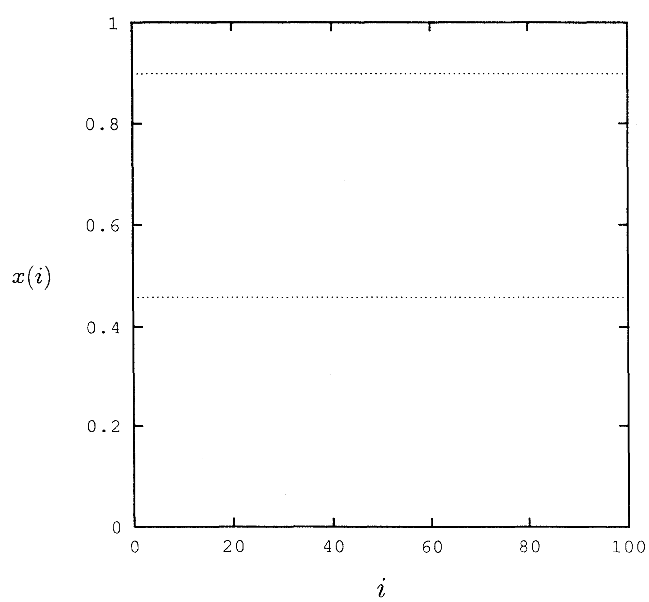

FIG. 1. The asymptotic spatiotemporal period-2 state of the system after the transient process at $a=4$, $\epsilon=0.15$, and $L=100$.

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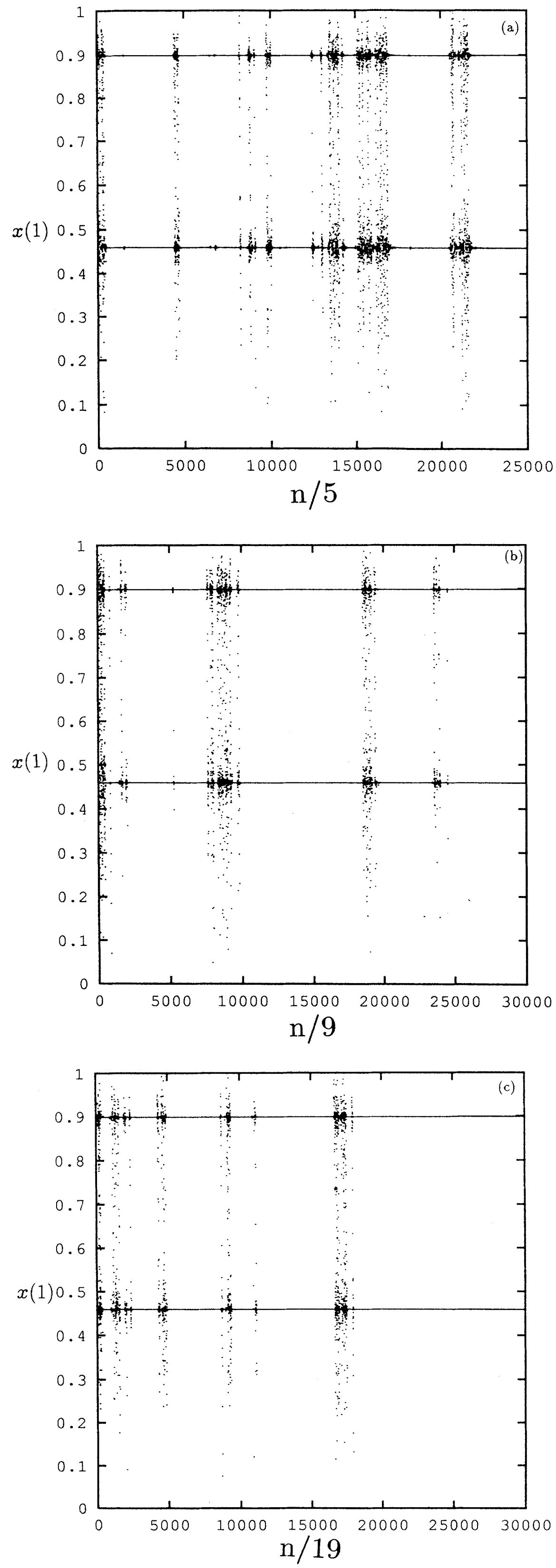

FIG. 2. Some supertransient evolutions of the first site. (a) $L=100$; (b) $L=500$; (c) $L=1000$. The style of on-off intermittency is easily observed in these figures.

process is very long, and exhibits a very interesting phenomenon, a characteristic of on-off intermittency, each site stays at the "off" state (defined as S2T2) for a very long time, and suddenly departs quickly from, and returns quickly to, the "off" state before the system is settled down at the S2T2 state. This feature is shown in Fig. 2, which shows the first site's transient processes with different system sizes of $L=100$, 500, and 1000. The space-time evolution of the system during a transient process is also shown in Fig. 3 with $L=100$. This figure is made as follows. If the site does not stay at the S2T2 state, then the corresponding pixel is black, otherwise, it stays white. The figure shows that the whole system is characterized by on-off intermittency during the transient process.

To further investigate the characteristic features of the transient on-off intermittency motion, we calculate numerically the average distribution $\langle P_n \rangle$ of the laminar phase shown in Fig. 4 for $L=100$, 500, and 1000. P_n represents the probability of the laminar phase of length n, namely, $P_n=M_n/N$, with N being the total number of segments of the laminar phase, and M_n the number of that of length n. $\langle\ \rangle$ means the average of all sites of the system, instead of that of the random initial conditions. These scaling curves show a perfect $-\frac{3}{2}$ power law as the system size becomes large.

In conclusion, we would like to make some interesting comments as follows.

As the system size L becomes large, many attractors or patterns coexist in the system, but their corresponding basins are very narrow, and their basin boundaries become very complicated and fractal [7]. When the initial conditions are randomly chosen, the system goes in a complicated fashion around these fractal basin boundaries for a very long time, and then finally is settled

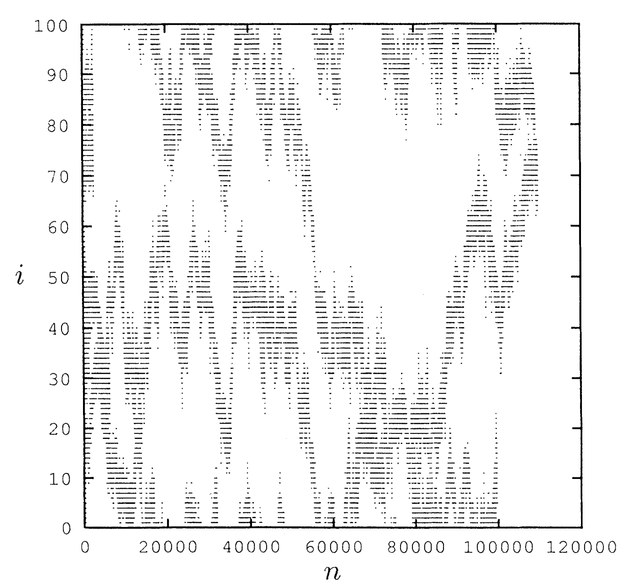

FIG. 3. The space-time evolutions of the system during a transient process at $a=4$, $\epsilon=0.15$, and $L=100$.

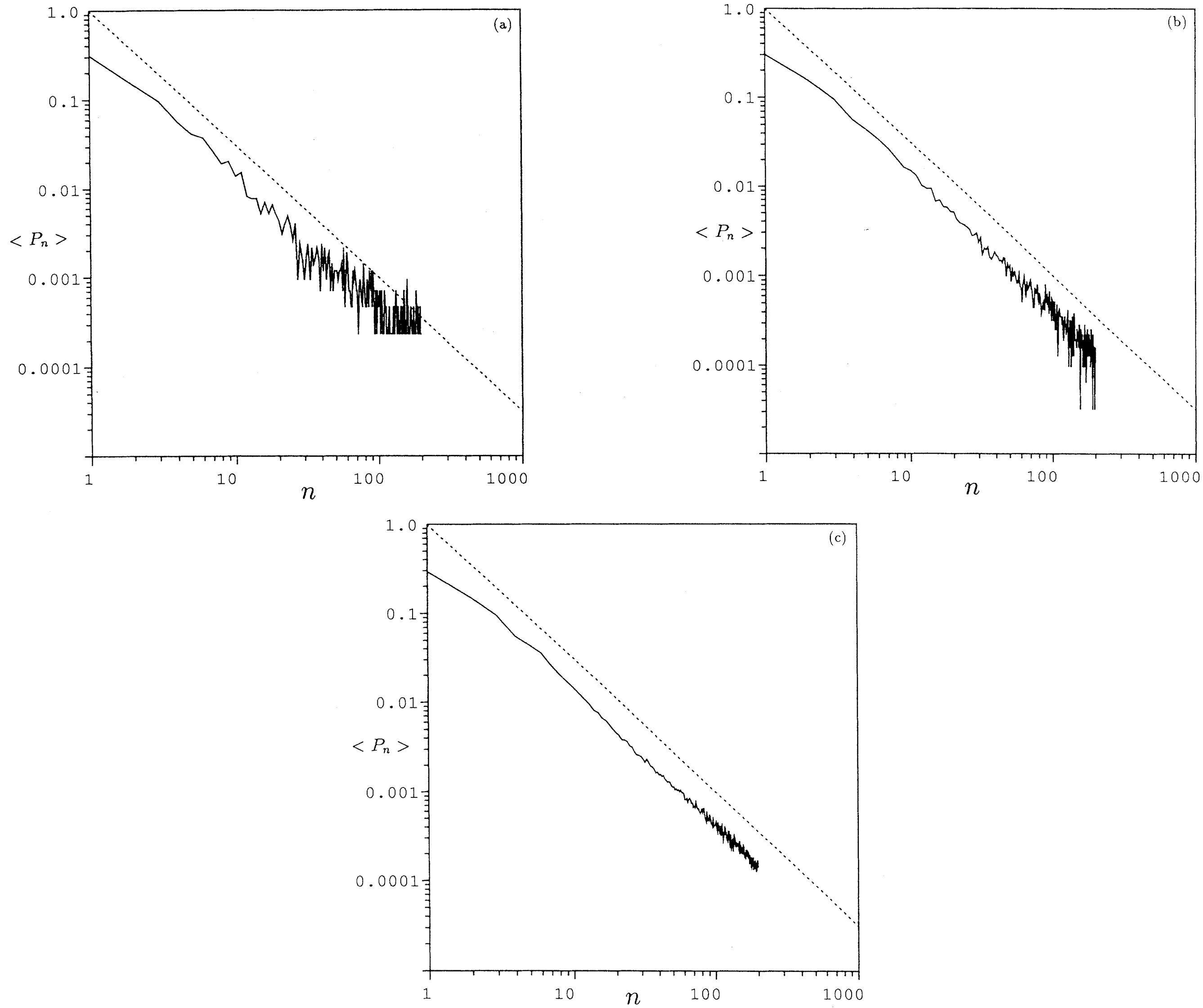

FIG. 4. The average distribution probability $\langle P_n \rangle$ of a laminar phase plotted against n (log-log plotting) for different system size. (a) $L=100$; (b) $L=500$; (c) $L=1000$. The dashed line is a perfect $-\frac{3}{2}$ power law decay.

down at an attractor. Thus, the system exhibits very rich and diverse transient behaviors.

In the parameter plane (a,ϵ), there is a region where the final state of the system is a spatiotemporal period-2 state with even L. The average transient lifetime of this state increases exponentially with system size L, and this transient process displays the on-off intermittency phenomenon, and obeys a power-law decay with exponent $-\frac{3}{2}$. However, as L is odd, it does not mate with the periodic boundary condition, and there is always one site which cannot at the S2T2 state. The site simulates the whole system to exhibit on-off intermittency permanently.

Actually, when we use CML to describe the spatially extended systems, we should make the system size L very large, precisely speaking, infinity; then the behavior of the system is not affected by the periodic boundary condition, and the model can truly reflect the behaviors of actual physics spatiotemporal systems. Therefore, if L is taken to be large, the supertransient lifetime becomes long enough so that the observation of the asymptotic S2T2 state is practically impossible, and the system always displays the on-off intermittency phenomenon within our computational time.

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